Nothing Infinite: A Summary of *Forever Finite*

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Abstract: In *Forever Finite: The Case Against Infinity* (Rond Books, 2023), the author argues that, despite its cultural popularity, infinity is not a logical concept and consequently cannot be a property of anything that exists in the real world. This article summarizes the main points in *Forever Finite,* including its overview of what debunking infinity entails for conceptual thought in philosophy, mathematics, science, cosmology, and theology.

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1. Infinity As We Know It

Despite the lack of academic consensus on how infinity should be technically defined, we can say that infinity is lexically defined as the condition of being *infinite*. Since to be infinite is the opposite of being finite, and to be finite is to be limited, it follows that to be infinite is to be without limit. Infinity is therefore the condition of being *limitless* or *unlimited* while finitude is the condition of being *limited* [1]. This is all straightforward, but the analysis gets more complicated from here.

2. The Varieties of Infinity

Infinity is about being limitless in *measure*. But there is more than one way to measure things—one can measure by quantity, by quality, or some combination of both. Hence, there is more than one way to be infinite, or limitless—there is limitlessness of quantity, limitlessness of quality, and a combination of both. The first is *quantitative infinity*. The latter two are *qualitative infinity* and *absolute infinity*, respectively. **Figure 1** shows that quantitative infinity and qualitative infinity each divide into two different subcategories of infinity, while absolute infinity is a hybrid of one subcategory of quantitative infinity and one subcategory of qualitative infinity [2].

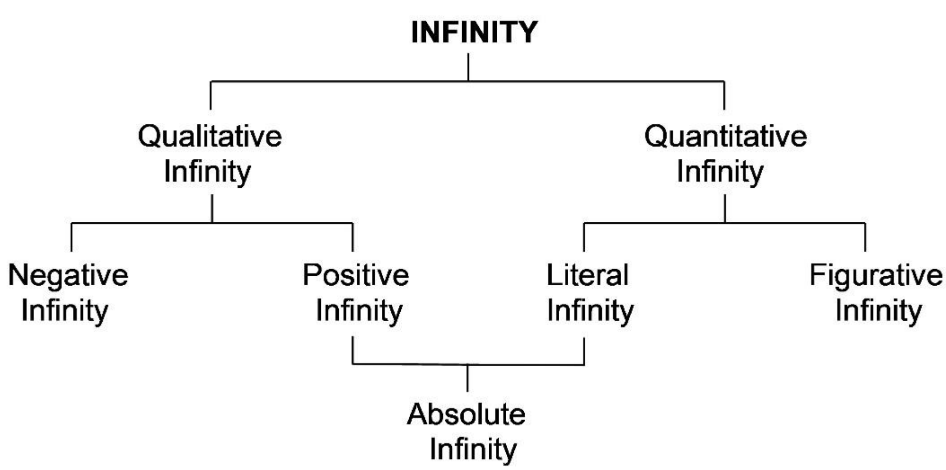

Figure 1: *The varieties of infinity.*

2.1 Quantitative Infinity

In measuring by quantity, such as measuring how many elements are in a collection of some kind, the measure is finite if the quantity is limited and infinite if the quantity is limitless. Quantitative infinity can thus be defined as *the condition of having a limitless quantity* [3]. This is the mathematical notion of infinity in its most general conception, independent of any particular mathematical procedure.

But quantitative infinity can be articulated in two different, but related, senses. Both senses agree that infinity is "the condition of having a limitless quantity," but one of these senses portrays such a condition literally while the other portrays it as only a figure of speech [4].

2.1.1 Literal Infinity

Philosophers and mathematicians have given the literal sense of (quantitative) infinity various names—*actual infinity*, *proper infinity*, *completed infinity*, *determinate infinity*—and yet other names, but the author prefers to use the term 'literal infinity' [5]. Literal infinity may be defined as *the condition of being both complete and limitless in quantity* [6]. To be literally infinite, a collection needs to have not only a limitless quantity of members but also a quantity of members that is complete, where both terms—'limitless' and 'complete'—are taken as literal in meaning.

2.1.1.1 Completeness

Mathematicians give completeness various technical definitions, each useful for performing a specific, mathematical operation. The completeness of a Cauchy convergence is not synonymous with the completeness of a Dedekind continuum, and neither of these conceptions of completeness is synonymous with the completeness of a Cantorian bounded set [7]. But the Cauchy convergence, the Dedekind continuum, and the Cantorian bounded set are all described as 'infinite' in the literal sense. Consequently, infinity in its literal sense is a more general concept, assumed regardless of any particular mathematical operation [8]. And since literal infinity is, in part, a condition of completeness regardless of the mathematical procedure to illustrate that completeness, we therefore need a more general definition of completeness to understand what it means to be literally infinite, independent of the particulars for showing completeness via any of the various technical, mathematical operations.

Completeness defined in the most general way, agnostic to any particular mathematical procedure or process, is simply the condition of being complete, where the word 'complete' means *to have all members necessary to be representative of a given class* [9]. Although that definition itself sounds rather technical, it can actually be seen to be a rather simple concept after we analyze its keywords: 'all', 'members', 'necessary', 'representative', and 'class'.

The word 'members' implies a collection of some kind. A collection is made up of members, such as objects or elements.

The word 'class' refers to a collection of objects sharing the same type of relation(s). Take a collection of objects such as hammers, saws, and screwdrivers. These objects all belong to the class of objects known as tools because they all share a common relation—they are all manual implements for performing work. A collection of people can also share a type of relation and so represent a class. A collection of students all belong to the same class if they share the same type of relation such as graduating in the same year.

The word 'representative' just means a good example of something. For a collection to be 'representative' of a given class—i.e., a good example of an instance of belonging to a class—the collection must be such that all its members share a common type of relation. For example, if each person in a collection of people plays music with all the others in the collection of people, they all share a “plays music” relation; if they are also professional musicians, they are, as a collective, representative of the class of people known as a music band.

The word ‘necessary’ refers in this context to a condition that must be met in order in order for any claim that x belongs to a class to be true. For example, a collection of musical instruments must include wind, string, brass, and percussion instruments in order for the collection to be representative of a class of objects known as a symphony orchestra.

Another keyword in the definition of ‘complete’ is the word ‘all’. In this context, ‘all’ means *each and every*. We know a collection is complete—we know it is representative of a given class—if *all* (each and every one of) the members in the collection are that are necessary for the collection to be representative of its class are in the collection. A collection can have members that are not necessary while still being representative of a class, but the collection must have all the necessary members if it is to be representative of the class—that is, if it is to be a *complete* collection.

The word ‘all’ in the context of completeness implies the collection referred to as complete has additional properties. Namely, the collection must be a *whole* (divisible but undivided), *entire* (without missing members), *finished* (all done forming), *full* (unchanging parameters at the time of consideration) *totality* (a collection having a total number of members, at least in principle). A collection is complete if, and only if, the collection is whole, entire, finished, full, and total. [10].

The word ‘complete’ in this sense applies to physical collections. A complete bag of marbles, for example, is a bag containing a whole, entire, finished, full, and total collection of marbles. This way of defining ‘complete’ also applies just as well to mathematical collections such as sets of numbers or geometrical figures.

Consider the set of the first six whole numbers: {0, 1, 2, 3, 4, 5}. The set is ‘whole’ because, while it is divisible, it is not divided in the sense of the members being separated into two or more collections such that the set depicted no longer has the necessary members related in such a way as to represent a single segment of whole numbers. The set is also ‘entire’ because no subsets—such as (2, 3)—are missing from it. The set is ‘finished’ because the process of constructing the sequence of numbers in the set (0–5) is all done. The set is ‘full’ in that no more numbers (such as 6, 7, 8, etc.) can be added to the set without changing the parameters of the set as being a set of only the first six whole numbers. Finally, the set is a ‘totality’ in that the set is a collection with a total: 6 represents the total—the exact sum—of *numerals* in the set while 5 represents the set’s total with respect to the cardinality of *numbers* making up the set [11]. Hence, {0, 1, 2, 3, 4, 5} as the set of the first six whole numbers is the whole, entire, finished, full, and total set of the first six whole numbers and thus the *complete* set of the first six whole numbers.

Now take this notion of completeness and transfer it from the finite to the infinite, for literal infinity is a condition of being complete as well as limitless. To be literally infinite, a collection must be a complete collection, as is, without further modification. If to the contrary the whole of a collection is *incomplete* simply qua collection, then there is some specifiable total to the collection which renders it finite. This is true even if the collection has a continuously running total of elements that grows ever larger in quantity. A collection of that sort is at best *figuratively* infinite rather than *literally* infinite.

Since a complete collection is a whole, entire, finished, full, and total collection, and since a literally infinite collection is a complete collection, then we know a literally infinite collection must likewise have these properties of completeness. For example, a literally infinite collection is a totality—it has a ‘total’ of some kind. For a literally infinite collection, having a total means the collection must be a non-finite ‘totality’—that is, the collection must have a total not equal to zero but also not finite—the collection must have a total number of members in the sense of having an *infinite number* of members [12].

There are various conceptions of what it means for a collection to have an 'infinite number' of members [13]. Some mathematicians hold that an infinite number is a number with infinitely many digits that can only be represented by a placeholder such as a letter of the alphabet or by a symbol such as the lemniscate (∞) as is typically used in algebra and calculus. Most mathematicians, however, follow transfinite mathematics in holding that an infinite number is a number greater than any finite number [14]. An infinite number in this sense is typically depicted by the Greek small omega (ω) or the Hebrew aleph (ℵ) and operates by mathematical rules different than those used for ∞ in algebra and calculus.

If infinity is literal infinity, then regardless of whether we consider infinity as ∞ or as the transfinite ω or ℵ, such symbols denote an 'infinite number' in the sense of a *limitless* number for the complete collection called 'infinite'. Limitlessness is, after all, the root meaning of the word 'infinity'.

2.1.1.2 Limitlessness

A limit is what specifies the 'range' (measurable extent of elements) of a collection, whether the collection is a set, sequence, series, etc. A limit may be an end, brink, border, bound, extreme, maximum, etc., as all such examples specify the range of a collection [15]. For a collection to be limitless implies there is no such condition—no end, brink, etc.—by which one can specify the range of the collection. To be limitless in the literal sense of the term is to have a range *unspecifiable, even in principle* [16]. One cannot specify (put in exact, definite terms such as by a definite, quantitative measure) the end, the bound, extreme, maximum, etc. of the collection said to be limitless.

2.1.1.3 Complete and Limitless

Some collections (sets, sequences, series, etc.) are commonly claimed to be infinite in the literal sense such as certain sets of numbers, geometrical figures, possibilities, and other abstractions but also certain collections of real-world objects like atoms, stars, galaxies, and some even say universes. Whether the members of the collection are abstract or concrete, the literally infinite collection is a complete (whole, entire, finished, full, and total) collection the quantity of members for which is also limitless (ergo, unspecifiable not just in practice but also in principle). The literally infinite collection is any collection said to have infinitely many members or to have at least two members between which there are infinitely many others, or to have at least one member that is infinitely far from another one where the use of the adverb 'infinitely' is intended to be taken at face value. Literal infinity is given various technical expressions in mathematics—we find instances of such in algebra, calculus, geometry, and transfinite mathematics [17].

2.1.2 Figurative Infinity

To be infinite in the figurative sense is to be actually finite and merely *seem* to be literally infinite. Philosophers and mathematicians have given the figurative sense of (quantitative) infinity various names—*potential infinity*, *improper infinity*, *incomplete infinity*, *variable infinity*—and others as well, but the author prefers 'figurative infinity' [18].

To be figuratively infinite, rather than literally infinite, is *not* to be a completed collection of components, all of which exist together at once in a final totality. Rather, figurative infinity is the condition of a necessarily *incomplete* collection in which changes to the collection's membership can continuously accrue while the collection never reaches a final quantity of members [19]. Figurative infinity may thus be defined as *the condition of indefinitely changing in quantity* [20].

Notice the word ‘indefinitely’. To be indefinite is *to have undefined or unspecified limits* [21]. The indefinite is not limitlessness but that which has unknown limits [22]. Despite the limits being unknown, having limits at all entails that indefiniteness is not a species of infinity but rather a species of finitude. Hence that which is finite can be either definite (having known limits) or indefinite (having unknown limits).

For example, a finite collection in which the members all exist simultaneously as a set of elements but which has so many elements that the collection’s quantity of elements is unknown, and perhaps cannot be known in practice, is an indefinitely large collection. The collection is still finite; it’s just too large to have known limits. Such a finite collection is a *collective indefinite* [23].

Now take another example. Consider a finite collection the members of which accumulate over time. The members of the collection may simply be a series of events in time or steps in a process, or the members may be objects that aggregate like a series of books. Suppose the series of events or objects increases persistently or ceaselessly but, no matter how long the series goes on, there is at any time only a finite number of members in the series. A series of this type goes on indefinitely but it does not go on infinitely—at least not in the literal sense of ‘infinite’—because at any time only a finite number of members in the series exists. Such a finite series is a *serial indefinite* [24].

Each serial indefinite is a series that continues indefinitely but still has a limit all right, just one that is unknown and so not apparent. The serial indefinite is a series without *apparent* limits. The serial indefinite is consequently a series that has only the illusion of being without limit at all. For that reason, serial indefinites are frequently referred to as ‘infinite’ series, which is a figure of speech to portray the serially indefinite as being without apparent limit [25]. Examples of serial indefinites referred to as infinite include instances of describing processes or progressions that “go on forever” or that “never end.” In actuality, the indefinite series remains finite no matter how big it grows or no matter how small an amount it diminishes—the serial indefinite is, therefore, a series that “indefinitely changes in quantity,” which is the definition of figurative infinity. Figurative infinity is thus another name for serial indefiniteness [26].

The lemniscate or “love knot” symbol (∞) is often taken as the symbol for figurative infinity. However, in general mathematics there are examples of ∞ used in the literal sense as well. Whether ∞ refers to literal infinity or to figurative infinity depends mostly on what the individual mathematician using ∞ has in mind by the symbol [27].

2.2 Qualitative Infinity

Quantitative infinity is just one kind of infinity—the mathematical notion of infinity. Another kind of infinity is qualitative infinity, which is one of the theological notions of infinity. Divine beings such as God are often described as ‘infinite’ in a qualitative, vice quantitative, sense of the term.

That is not to say the divine is never described in quantitatively infinite terms, for the divine is indeed so described. For example, consider some of God’s qualities such as omnipotence, omniscience, and omnipresence. These are typically defined in terms of infinity—omnipotence as infinite power, omniscience as infinite knowledge, and omnipresence as infinite presence. And the ‘infinite’ aspects of these qualities are sometimes described in the quantitative sense of infinity. The infinite power of God has been taken to imply God can do infinitely many things, the infinite knowledge of God is sometimes taken to mean that God knows infinitely many true statements, and the infinite presence of God is often interpreted to mean God is infinitely many places—in each case, the “many” in “infinitely many” is a quantitative infinity [28]. However, beyond these quantitative uses of infinity to describe divine qualities, there is another notion of infinity that tends to be attributed to divine beings such as God: qualitative infinity.

Qualitative infinity is *the condition of having unlimited quality* [29]. Quality is about value, importance, or performance. Something is qualitatively infinite—something has no finite measure to its quality—if it has a property that sets such a high standard that no finite property can compare in value, importance, or performance. There is no way even in principle to distinctly identify the conditions by which something of finite quality could be improved in value, importance, or performance to match the qualitatively infinite. The qualitatively infinite is of such superior quality as to be 'immeasurable' or 'incomparable' to the qualitatively finite. [30].

Qualitative infinity is a theological notion of infinity because theologians typically hold that God's qualities or attributes—power, knowledge, presence, etc.—are infinite in the sense of being qualitatively 'unlimited'—lacking limit in value, importance, or performance—rather than only quantitatively 'limitless'. However, theologians do not all agree on what it means for God to be qualitatively infinite. For there is more than one way of being qualitatively infinite. Both ways agree that the infinite nature of God is "the condition of having an unlimited quality," but one of these senses portrays such a condition in positive terms, the other in negative terms [31].

2.2.1. Negative Infinity

Some theologians believe the infinity of God is 'negative'—not in the mathematical sense or in the moral sense but rather in the sense that something is negated (i.e., denied). Negative infinity is *the condition in which unlimited qualities are indistinguishable* [32].

To see what this means, consider some qualities God is usually said to have such as wisdom, beauty, and power. Many theologians say these qualities are 'infinite'. God is said to be infinitely wise, infinitely beautiful, and infinitely powerful. In other words, God's wisdom is unlimited, God's beauty is unlimited, and God's power is unlimited. Now, if God's infinity is of the negative variety, with each of those qualities (wisdom, beauty, and power) being negatively infinite, then all of those unlimited qualities are indistinguishable from one another. It would be just as accurate to say the infinite wisdom of God is God's infinite beauty, that the infinite power of God is God's infinite wisdom, and so on. All distinctions between God's wisdom, beauty, and power break down due to their negative infinitude. To call God infinitely powerful may be to emphasize God's power, but technically there is no distinction between God's infinite power and God's other infinite qualities. Moreover, God's (negative) infinity implies a doctrine of *divine simplicity*: God's very being is just a single infinite quality that only appears to our human perspective as different kinds of qualities [33].

2.2.2. Positive Infinity

Not all theologians believe God's infinity is negative. Some say quite the opposite, that God's infinity is positive. In this context, the word 'positive' is not mathematical or moral in meaning but rather qualitative in meaning—to be positive is to affirm something about a quality rather than to deny (or negate) something about a quality. To say that God has infinite qualities is to affirm the infinitude of each of those qualities rather than deny that the qualities are distinct qualities owing to their infinitude. In other words, the positive nature of infinity does not negate distinctions between qualities that are infinite.

Another way to put it is that each quality that is infinite in the positive sense of 'infinite' is a distinct quality from any other quality said to be infinite. A quality that is positively infinite is a quality affirmed as infinite—as unlimited—all on its own, rather than indistinct from any other infinite quality. Positive infinity is *the condition in which distinguishable qualities are unlimited* [34].

For something to be positively infinite (or unlimited) is for it to be *incomparable* with any other finite instance of the same thing. That is not to say a quality that is positively infinite is absolutely incomparable with finite qualities such that the infinite quality cannot even be recognized for the quality that it is; rather, it is just to say the quality is relatively incomparable such no finite instance of the same quality is up to the same standard. For example, infinite beauty is still beauty even though infinity beauty is incomparable to finite beauty; infinite knowledge is still knowledge even though infinite knowledge is incomparable to finite knowledge; infinite power is still power even though it is incomparable to finite power, and so forth [35].

God, being positively infinite, has various attributes (beauty, knowledge, power, benevolence, and so forth) that are each positively infinite in the sense of being incomparable—God has incomparable knowledge, incomparable presence, incomparable power, and so on. Contrary to the notion of negative infinity, the positive infinity of God's attributes does not render the attributes indistinguishable from one another, even though they are each without qualitative limit [36]. It's just that nothing of finite quality can compare to something of infinite quality.

2.3 Absolute Infinity

This is another theological notion of infinity. A minority of theologians believe that the infinity of God is not a negative infinity but it is also not purely a positive infinity either. Instead, God's infinity is infinity both in the sense of literal (quantitative) infinity and positive (qualitative) infinity, together as a single form of infinity called *absolute infinity* [37].

Absolute infinity is the unlimitedness of quality that emerges from, or results from, something of unlimited quantity. When something is of unlimited quantity, it is qualitatively different from any finite instance of that quality. Absolute infinity is *the condition of having unlimited quality from a lack of quantitative limits in measure* [38].

To get a handle on absolute infinity, start by considering any one of God's attributes, such as God's knowledge. For God to have infinite knowledge means, in part, that God knows infinitely many things (this is the quantitative aspect of God's infinite knowledge); but for God to have infinite knowledge also means God's knowledge of any one thing is of incomparable quality to finite knowledge of the same. So God has absolutely infinite knowledge because God's positive infinitude of knowledge is a result of God not only knowing infinitely many things but also because of the infinite (incomparable) quality with which God knows those infinitely many things. God's knowledge is qualitatively infinite because of the limitless quantity of things God knows, but God's knowledge is also not qualitatively reducible to the quantity of what God knows. God's infinite knowledge is thus an instance of absolute infinity—it is a positive infinity emergent from and irreducible to the quantitative infinity of what God knows, a kind of perfection [39].

Moreover, theologians who believe God's infinity is absolute typically hold that not only are each of God's qualities or attributes independently infinite (as in absolute infinite knowledge, absolute infinite power, absolute infinite goodness, etc.) but also God is absolutely infinite because God has some quantitative and qualitative "essence" that is absolutely infinite independent of any of God's other qualities, making God as such an instance of absolute infinity—God, in this view, just *is* Absolute Infinity [40].

2.4 Divine Infinity

Both qualitative infinity and absolute infinity are versions of what some theologians refer to as *divine infinity* or the infinity of God [41]. Divine infinity stands in contrast with quantitative infinity. Hence,

- quantitative infinities: *literal infinity* and *figurative infinity*.
- divine infinities: *qualitative infinity* (positive and negative) and *absolute infinity*.

The bulk of *Forever Finite* deals with quantitative infinity (especially literal infinity) rather than divine infinity and so the qualitative infinities and absolute infinity are addressed in less detail, but each is given its due attention.

3. The Charge Against Infinity

Forever Finite makes the case that the concept of infinity—the concept of an affirmative property without limitation—is intrinsically erroneous [42]. This holds for all the varieties of infinity in **Figure 1**, which includes both the quantitative infinities and the divine infinities.

4. Basic Assumptions of the Case

In making the charge that infinity is an erroneous concept, the author makes a few assumptions about infinity [43], including the following:

- Infinity can be defined.
- There are various kinds of infinity.

These are safe assumptions. The previous sections define infinity and offer an explication of the various kinds of infinity. If there is an additional assumption here, it is that the author also assumes these are all the relevant senses of the word ‘infinity’ and its cognates, which is perhaps debatable but nevertheless plausible.

In addition, there is one more assumption the author makes and it is key to the case against infinity:

- Infinity of any kind must at least be logically coherent in order to be a reality.

In charging that infinity is an erroneous concept due to logical inconsistencies intrinsic to any relevant understanding of infinity, the author assumes all concepts should be logically coherent (that is, not in violation of logic) if they are to avoid being erroneous. In other words, concepts must not be intrinsically illogical if they are to stand any chance of providing one with a reliable understanding of the subject to which they are predicated and refer to anything that exists in the real world. That includes infinity [44].

Not all philosophers agree with the third assumption. Some philosophers take the contrary point of view, asserting that some things are intrinsically paradoxical or logically ‘paraconsistent’ (able to violate certain logical principles) and that infinity is one of those paradoxical or paraconsistent things [45]. The author pushes back, arguing against such a position as a fallacious appeal to mystery [46].

Consequently, any property claimed to be true of something must not be conceived in such a manner as to imply logical inconsistencies; otherwise, the claim cannot be taken as accurate and should be rejected [47]. Infinity is such a property, for it is a property commonly claimed to be true of certain collections (for quantitative infinity) or certain attributes (for divine infinity). But because the very concept of infinity is riddled with logical inconsistencies, infinity is an erroneous concept that cannot be accurately applied as a description of anything that actually exists and so the use of infinity to describe anything should be rejected [48].

5. The Case Against Infinity in Brief

Anything that is a mere absence, an emptiness, or that is devoid of content, is technically speaking without limit. Take zero (0) for instance. What is the limit of zero? The answer is moot, for while zero can *be* a limit, zero does not *have* a limit [49]. And yet, we would not want to say that zero is “limitless” or “unlimited;” we reserve such words for that which is not absent of quantity or quality, that which is not devoid of properties we can affirm—in other words, for the infinite.

Consider as well a simple circle. The curvature of the circle has no limit—at least not in the form of a break, border, or boundary. And yet we would not describe the curvature of a circle as “limitless” or “unlimited,” except perhaps as a figure of speech. Instead, we would use a narrower term like ‘unbounded’ to specify the kind of limit that is lacking for the curvature of the circle. After all, the circle is not without limit *per se*. A circle’s closed curvature does have a limit with respect to its measure—its circumference—as can be seen by tracing the circle’s curvature all the way around back to the same mark (**Figure 2**) [50].

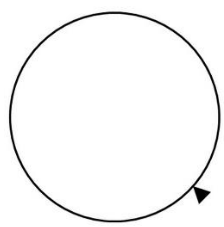

Figure 2: *The circumference of a circle is finite as indicated by tracing around the circle from a mark along its curvature.*

Words such as ‘limitless’ or ‘unlimited’ are thus not usually applied to everything lacking a limit of some particular kind—things such as zero or the empty set, which has nothing to be limited, or the curvature of a circle which is geometrically closed without borders or breaks. Rather, words such as ‘limitless’ and ‘unlimited’, as used in their literal senses, are more often reserved for that which is regarded as literally infinite.

The author contends that, while lacking a limit *per se* is not a logical problem, such is not the case with infinity. Infinity is the condition of being limitless or unlimited with an affirmation of *measure*. The very concept of infinity, the very concept of being limitless or unlimited in measure, is riddled with logical troubles with respect to both discursive and practical reasoning.

In terms of discursive reasoning, the logical troubles are a matter of what infinity *means*—its meaning implies *self-contradictions*. In terms of practical reasoning, the logical troubles intrinsic to infinity are a matter of how the word ‘infinite’ is *used* in describing things—the word is such a misnomer that its use to prone to mislead one in understanding what is described as ‘infinite’. The problem with the concept of infinity can thus be summed up quite simply [51]:

- The degree to which infinity is not a self-contradictory concept is the degree to which it remains a misnomer for certain finite properties and thus the degree to which infinity is a misleading term prone to inconsistent usage.

Self-contradiction and an intrinsic tendency to mislead are violations of logical reasoning—both discursive reasoning and practical reasoning. The author of *Forever Finite* contends that infinity implies illogic of both sorts and so infinity in all its varieties must be rejected as conceptually erroneous, however popular and useful infinity has proven to be.

5.1 The Case Against Literal Infinity

The term ‘infinite’ as applied to a given collection (whether set, sequence, series, etc.) denotes the collection in question is in a state or condition of being complete and limitless in quantity, where ‘complete’ and ‘limitless’ are both taken quite literally. Consider the numbers zero (0) and one (1). Both are whole numbers, but they are also real numbers. The standard view in mathematics is that between any two real numbers is a limitless sequence of other real numbers—all the decimal numbers between 0 and 1. So, what we apparently have here is a ‘set’ of numbers in which not only is the quantity of elements comprising the set limitless, but the set is also complete with both a beginning number (0) and an ending number (1). Complete and limitless—literally infinite.

Or so it seems. According to the author, the logical coherence of literal infinity is only an illusion. Literal infinity is actually a self-contradictory concept. The contradiction hidden in the concept of literal infinity is between the implications of what it means for a collection to be complete and the implications of what it means for a collection to be limitless. If it is contradictory to be both complete and limitless, then no collection—and so no set of numbers—can be literally infinite.

Forever Finite reveals the contradiction by considering the implications of completeness and the implications of limitlessness, then comparing and contrasting the two sets of implications. The following is a brief overview of the book’s account of the contradictory implications.

An infinite collection is a complete collection. We know from § 2.1.1.1 above that the completeness of any collection implies the given collection is whole, entire, finished, full, and has a totality of elements. So, an infinite collection, as a complete collection, must also have these properties.

As a complete collection, an infinite collection is a *full* collection. But for a collection to be full, it must have a maximum quantity of elements. Ergo, an infinite collection must also have a maximum quantity of elements—albeit an infinite maximum. That infinite maximum is a *totality* of elements—an infinite totality or infinitely large total of elements. Totals are countable at least in principle if not in actual practice. An infinite totality must therefore also be countable (though *only* in principle). Moreover, as a complete collection, any count of all the elements in the infinite collection must cover the *entire* collection. A count of that entirety is equal to an infinitely large number of elements. That infinitely large number is not a running total but a standing total, for the complete collection is a *finished* collection that must have (albeit only in principle) a last element to count that makes the count conclude with an infinitely large number. That infinitely large number is also the infinite sum of elements equal to the *whole* of the collection [52].

Literal infinity is not just a state of completeness though, for the very etymological meaning of the word 'infinite' refers to a condition of being without limit—*in* ("not") + *finite* ("limited"). An infinite collection is a limitless collection. As a limitless collection, however, the infinite collection entails just the opposite features of completeness.

The concept of being 'full' does not apply to that which has no limit and so there can be no maximum to a quantity without limit. By lacking a maximum of elements, a collection that is limitless is a collection that cannot be quantified in the sense of being given a total—such a collection has no 'totality'. It's not that the limitless collection fails to have a total; rather, it's that the concept of totality does not apply to that which is limitless. Furthermore, by lacking totality, the limitless collection cannot be counted. The limitless collection has no last element, no highest number, to count. And without a last element to any count or sequencing of elements, there is no concluding element to the limitless collection. And without a concluding element to count, the notion of being 'finished' does not apply to the collection that has no limit. While none of that means we cannot speak of the 'entire' or 'whole' of a limitless collection, it does mean that the properties of entirety and wholeness for a limitless collection are not the same as they are for a complete collection. The entirety of a limitless collection is not the kind of entirety captured by an *infinite number* of elements, but rather the entirety of a limitless collection can only be said to have *infinitely many* finite elements for which there is no "infinite number." As for the whole of a limitless collection, that whole is not *equal* to a so-called "infinite sum" of elements comprising the collection but rather the infinite whole is *greater* than any sum comprising the collection and there is no such thing as a limitless sum [53].

Clearly, the implications of a collection being quantitatively limitless are in direct contradiction to the implications of the same collection's properties of fullness, totality, entirety, finish, and wholeness—in other words, the collection's completeness. What these contradictory implications show is that the very concept of being both complete *and* limitless in quantity—that is, the concept of being *literally* infinite—is inherently self-contradictory, not merely "paradoxical" [54].

Now, if literal infinity is genuinely self-contradictory, you might wonder why no one has pointed it out before now. Actually, many philosophers and mathematicians over the millennia have noted genuine contradictions implied by the concept of literal infinity [55]. Even so, only a minority rejected literal infinity outright. Most ignored the contradictions, waving them off as paradoxes caused by the vagueness of pre-theoretic language or the inability of the human mind to comprehend infinity [56]. Either approach provided a rationale for infinity to be retained, though each rationale is rather thin and unconvincing.

Eventually, a few mathematicians in the 19^{th} and early 20^{th} Centuries made serious attempts to "tame" infinity—to either solve or dissolve literal infinity's self-contradictory implications [57]. One mathematician to take up that challenge was Georg Cantor (1845–1918). He proposed a new system of mathematics for quantitative infinity, which he termed the *transfinite* in order to distinguish quantitative infinity from the divine infinity of God. Cantor intended his transfinite set theory and transfinite mathematics to make literal infinity (or 'actual infinity') a logically and mathematically coherent concept [58].

To this day many, if not most, mathematicians assume Cantor succeeded. But that is not so. Cantor only half succeeded. He did manage to invent a mathematically consistent system, but *mathematical* consistency and *logical* consistency are two different things. While Cantor's transfinite system is, at least for the most part, mathematically consistent, there are nevertheless gaping logical flaws in it that have to do with the meaning of infinity and the rationale for predicating infinity to number systems [59].

For one thing, Cantor proposed his infinite or 'transfinite' numbers (ω and ℵ) to represent literal (actual) infinity, but that turns out not to be so. Cantor redefined in technical terms what it means for an infinite collection to be "complete" and "limitless." His technical redefinitions of these properties were intended to ensure the completeness and limitlessness of infinite sets would not carry contradictory implications. However, Cantor's redefinitions of infinity and its constituent properties of completeness and limitlessness entail that his system operates with what are in reality only *figuratively* 'infinite' quantities, not *literally* infinite quantities as originally claimed. Which is why Cantor's system works mathematically—it isn't really operating with literal infinity at all, but rather a faux 'infinity' passed off as if it is literal infinity (what Aristotle called 'actual infinity') [60].

There are some further flawed assumptions underlying the reasoning behind Cantor's system. He proposed his infinite numbers (ω and ℵ) represent an 'infinite set' of finite numbers. The natural numbers: {1, 2, 3, 4, 5, …} are allegedly such an infinite set. However, Cantor just takes the set-hood of number systems as an axiom while it is hardly self-evident. We could just as well assume the natural numbers comprise not a complete *set* of numbers at all but merely an incomplete (temporal) *series* under construction, where the ellipsis in the above sequence just means to keep on inventing more, higher numbers as needed. If numerical systems are not sets but merely open series constructed by a rule—at best, a figurative infinity—then Cantor's entire system falls apart [61].

The logical problems with Cantor's system only get worse from there, as the author further details in *Forever Finite* [62]. Cantor's attempt to tame infinity fails—literal infinity remains just as self-contradictory as it did before the transfinite system was proposed.

The implications of literal infinity turning out to be self-contradictory are profound. Since we know that self-contradictions cannot manifest in reality, and literal infinity is intrinsically self-contradictory, then there cannot exist literal infinities in reality any more than there can exist square circles or married bachelors. Illogical concepts cannot manifest in the real world, and literal infinity is one of those illogical concepts [63].

If this is so, then we should find further logical contradictions in thought experiments involving the application of literal infinity to collections of real-world objects. In fact, we do.

Logical self-contradictions are implied, for example, by the notion of hotels with infinitely many rooms or the notion of libraries with infinitely many books [64]. Moreover, logical self-contradictions result from proposals of infinite space, infinite time, infinite states of motion, and the use of infinity in physics and cosmology [65]. In every instance, the self-contradictions entailed by such notions of the literally infinite are between infinity's constituent properties of completeness and limitlessness. Thus infinity, at least in the literal sense of the term, cannot be applied to the real world without implying logical contradictions. We can only conclude there is no literal infinity in the real, physical world.

If there is any hope for a rational account of infinity, it won't be found in taking infinity as a condition of being both literally complete and literally limitless in quantity. But, as we shall see, figurative infinity is not in much better shape, for it is too dependent on being the counterpart of literal infinity.

5.2 The Case Against Figurative Infinity

The word 'infinite' is sometimes used figuratively for series that appear to be limitless but which are actually finite. A figuratively infinite series is a series that continues indefinitely while remaining finite at every step along the way, having at any given time a limited quantity of steps taken, but increasing in the quantity of those steps as the series continues. A series of this kind is in more literal terms a ‘serial indefinite’ [66]. The term ‘infinite series’ is sometimes used for serial indefinites since such series give the illusion of having “no end” or “going on endlessly.” Referring to a serial indefinite as infinity or as an infinite series may thus be taken as a mere figure of speech [67].

Infinity is sometimes used figuratively in mathematics when a line is said to be “infinitely divisible,” meaning we may continue dividing the line into ever smaller segments as long as we wish. Another example of figurative infinity is a given period of time—such as a second—when it is said to be “infinitely divisible” into ever shorter increments of duration, measured to ever greater precision while nevertheless remaining finite in duration [68].

The figurative use of the word ‘infinity’ to denote the serially indefinite, as when an indefinite process is called ‘infinity’ or as when an indefinitely progressing series is said to “proceed to infinity,” ends up being a *misnomer* since indefiniteness is contrary to infinity. A serial indefinite, unlike literal infinity, is neither complete nor limitless. Rather, a serial indefinite is always incomplete and it does have limits—namely, limits that, if they could be measured, would indicate only running totals of steps that have been taken in the continuous formation of the series.

The use of misnomers per se is not necessarily problematic. Take the word ‘atom’ as it is used in physics and chemistry for the basic units of matter composing molecules. The word ‘atom’ originally referred to that which is *indivisible*, but what scientists called “atoms” were eventually shown to be divisible into parts—the atom was split into *subatomic* particles. The misnomer ‘atom’ stuck for the basic units of matter that compose molecules, and the word ‘atom’ has since taken on a meaning in physics and chemistry that no longer connotes indivisibility. The word ‘atom’ as used for the basic units of molecular composition can be used without worry of causing confusion, despite the word technically being a misnomer with respect to its etymology. However, not all misnomers are so benign. Some misnomers are indeed intrinsically misleading.

The word ‘infinity’ as used figuratively for the serially indefinite is just such a pernicious misnomer [69]. Use of the word ‘infinity’, while perhaps intended as a figure of speech for the serially indefinite, is a pernicious misnomer because both ‘infinity’ and its cognates ‘infinite’ and ‘infinitely’ are intrinsically misleading. Such words are inherently prone to *slippage*—that is, they all too often slip from being figures of speech into literal usage, thus confusing the two senses. Words such as ‘infinity’, ‘infinite’, and ‘infinitely’ when intended in the figurative sense too often are inadvertently used in such a way as to imply a condition of being complete and limitless in quantity—an instance of literal infinity. Hence, an unintentional slip in meaning from figurative to literal [70].

A good example is saying that something goes “to infinity,” such as when an exponential curve in calculus proceeds to ∞. Saying such may be a figure of speech for indicating that the curve can be extended *indefinitely* along a given dimension. However, saying an exponential curve goes to infinity often slips from a figure of speech that means the curve can be extended indefinitely through a dimension to meaning that the curve reaches a “point at infinity,” as if infinity is a destination—a complete and limitless distance away [71]. Likewise, calling a series “infinite” in the figurative sense of being indefinite in iteration often slips into talk of the very same series containing an “infinite number” (a complete and limitless quantity) of iterations—i.e., a literal infinity of iterations [72] Consider as well talk of there being “infinitely many” divisions for a line segment. Such a description often slips from meaning there can be indefinitely many divisions made along its extent to meaning there *are* infinitely many (a complete and limitless number of) places where the line segment is divisible—a literal infinity of divisible places in the segment [73].

Infinity in its figurative sense is thereby used inconsistently in practice, with the literal sense of infinity often erroneously implied [74].

Because infinity in its figurative sense is not a consistently used term, prone to too much slippage, the term ‘infinite’ should be avoided. It’s not that figurative infinity violates discursive logic; it’s that it violates practical logic. Terms that are prone to such slippage, terms that are prone to mislead, are not terms that send the message intended to be sent. If infinity and its cognates reduce to misleading misnomers, then mathematicians and physicists in particular are better off getting rid of such terms since they intend their fields to be based on precision. In place of the word ‘infinite’ more accurate terms such as ‘indefinite’, ‘persistent’, and ‘ceaseless’ should be used, with indefiniteness replacing the very concept of infinity altogether [75].

5.3 The Case Against Divine Infinity

Theologians typically describe as infinite various attributes of God, such as God’s knowledge or power, or they describe as infinite the ‘essence’ of God. These descriptions are instances of divine infinity, the conception of which, whether positive or negative or absolute, has always resulted in conceptual problems [76].

Divine infinity has some of the same conceptual problems as quantitative infinities. Literal infinity implies contradictions; so too does divine infinity. Figurative infinity is a misleading misnomer for certain finite qualities, and so it is with divine infinity [77].

As to the self-contradictions in the concept of divine infinity, they are found both in the negative and positive versions of divine infinity. Take negative infinity. For a quality of God to be negatively infinite is for that quality to be *indistinguishable* from God’s other unlimited qualities, including contrary or even logically contradictory qualities [78]. For example, if God is infinitely just and infinitely gracious, where both are one and the same, then for God to infinitely punish is for God to show infinite mercy and for God to show infinite mercy is for God to infinitely punish, which is nonsense [79]. Now consider positive infinity. Each of God’s qualitatively infinite attributes is distinguishable from the others, but there are still logical problems when such qualitatively infinite attributes are described in quantitative terms. For example, some theologians describe God’s infinite knowledge as God knowing infinitely many things all at once—that’s an instance of literal infinity and we’ve already seen that literal infinity is a self-contradictory notion [80].

The same problem afflicts divine infinity in the form of absolute infinity, the qualitative infinity that emerges from quantitative infinity. Because the qualitative aspect of absolute infinity depends on quantitative infinity and the quantitative infinity is literal infinity in particular, the same contradictions arise. God knows infinitely many things, can be infinitely many places, can do infinitely many things, etc.—all of which entail literal infinity, which is itself riddled with contradictions as previously described [81].

In rejoinder, many theologians claim God’s infinity is mystical or paraconsistent. They often say God’s infinity is “beyond human comprehension,” and even go so far as to say God’s infinity is only ‘paraconsistent’ in logic or even beyond logic altogether [82]. In other words, God’s infinity is mystical in nature. The author pushes back, arguing that appealing to mysticism is a version of the logical fallacy of appeal to mystery and that appeals to paraconsistent logic are empty rhetorical strategies that can bail any speculation out of a self-contradiction [83].

To avoid such illogic requires either taking divine infinity as merely figurative or rejecting it altogether as unfounded [84]. The former option regards calling God’s attributes “infinite” as metaphorical, a figure of speech for the unfathomably great (but still literally finite) quality of

God's attributes [85]. For example, to say, "God is infinite" simply means in more literal and precise language, "God has superlative attributes." Or to say some attribute of God is infinite, as when it is said that God has infinite knowledge, would be just to say that the given attribute is superlative, supreme, incomparable, or the like [86]. While this option is certainly better than abiding self-contradictory conceptions of the divine, it is nevertheless problematic. For if God's attributes are merely "infinite" in a figurative sense, we can always ask if it would not be both clearer and more accurate (not to mention more honest) just to say God's attributes are superlative, supreme, or incomparable and just leave it at that. There is no need to use a misleading misnomer like 'infinite' to refer to God's superior attributes. Consequently, divine infinity has the same trouble as figurative infinity—both are misleading terms, and so it would be better to likewise lay aside divine infinity in favor of more plausible, finite conceptions of divine attributes, just as it would be best to abandon the use of infinity in its figurative sense in favor of less misleading terminology such as indefiniteness, ceaselessness, etc. [87].

5.4 The Bottom Line in the Case Against Infinity

Infinity is riddled with logical inconsistency—both in terms of discursive logic and in terms of practical logic. Insofar as the quantitative infinities and the divine infinities are not self-contradictory, they render the word 'infinity' a misleading misnomer for what is more precisely and accurately finite in nature, which in turn undermines a clear understanding of the subjects to which infinity is so cavalierly applied. References to infinity should therefore be replaced by more appropriate terms.

6. The Verdict

The author acknowledges the reader will have to come to their own verdict but maintains that, in the case of logic versus infinity, infinity has lost the case. Infinity is guilty of being intrinsically illogical and, consequently, the concept of infinity is unable to refer to any existing state of affairs in the real world. Finitude is the nature of *being* as such—the nature of existence itself; to coin a slogan: *esse est finitus*—to be is to be finite [88].

7. Implications

Debunking infinity carries a variety of implications, some trivial and some profound. There are implications for common discourse, implications for conceptual thought in fields such as philosophy, mathematics, physics, cosmology, and theology, and there are implications for belief in immortality [89]. *Forever Finite* offers a brief overview of these implications.

With regard to common discourse, widespread acceptance of infinity's debunking would only slightly impact ordinary, colloquial speech. Even if referring to infinity or calling things infinite were to go out of style as a result of infinity's refutation, we would continue referring to certain finite things as being without a particular kind of limit, such as when we refer to the closed curve of a circle as being "unbounded" or when we refer the immense, the minuscule, the protracted, the ephemeral, the ceaseless, the persistent, the inexhaustible, and the unrestricted for what they are—*indefinite* in size or succession rather than infinite [90].

The implications of the case against infinity are more serious for philosophy. Because the literal sense of infinity implies logical contradictions, philosophers should not expect logical outcomes from paradoxes involving infinity. Since philosophers pride themselves on being logically precise, they should replace the use of infinity and instead make use of more precise alternatives [91].

For mathematics, the case against infinity carries implications that impact the field to various degrees. However, even if the case against infinity were widely accepted by mathematicians, such would not cause a crisis in their field if for no other reason than some mathematicians already believe nearly all mathematics can be framed in finite terms [92]. The greatest change to mathematical practice would be a change to terminology and syntax rather than to the operations in arithmetic, geometry, algebra, and calculus. If the case against infinity is sound, then mathematicians should replace the use of infinity in their field of study with the use of more accurate concepts such as the concept of 'indefiniteness' and adopt new notation for such [93]. The operations in general mathematics that previously relied on infinity can, however, go on as usual even if infinity is replaced by indefiniteness and even if the symbolism and notation once used for infinity were to be replaced with new symbolism and notation for indefiniteness. A rather trivial tweak to mathematical practice. That said, rejection of infinity would also entail something more significant for mathematics—revision to the conceptual framework assumed for mathematical foundations (see § 8 below).

More serious still would be the impact of rejecting infinity on disciplines such as physics and scientific cosmology. If quantitative infinity is a logically inconsistent concept, then physics and cosmology must reject all theories of infinite physical magnitudes such as infinite expanses of space, infinite durations of time, infinite amounts of energy, and so on. Everything from certain versions of the Big Bang theory to various speculations about infinitely many universes making up a so-called 'Multiverse' would consequently require revision or replacement. Basically, replacing infinity would become necessary for physics and cosmology to improve the accuracy of scientific models of the Universe [94].

With respect to the future of theology, the author acknowledges that most theists are unlikely to be persuaded by the case against infinity. This is largely due to the weight of religious tradition and worries that a conception of God as finite would undermine belief in the divine. *Forever Finite* maintains that such worries are overblown and goes on to argue that construing the divine as finite would actually lend needed credibility to any theology. [95].

Finally, with regard to hopes for immortality, debunking infinity impacts only certain notions of immortality, but certainly not all such notions. Specifically, the idea that the future is literally infinite would have to go and so personal immortality over a literally infinite future is not feasible. But that leaves the possibility of personal immortality as either 'open-ended life' over a ceaselessly growing finitude of future time or 'immutable life' in a static spacetime block, or 'timeless life' from a point of view somehow beyond time altogether. These versions of immortality remain free of refutation by the case against infinity. That is not to say such ideas are necessarily logical, just that the case against infinity has no implications for them [96].

8. Further Research

Forever Finite has both a print edition (publication date: August 2023) and an expanded, online edition (to be published December 2023). The online edition includes an appendix offering a new conceptual framework for a finite mathematics based on the concept of indefiniteness [97].

However, the conceptual framework is just that—a framework for a theory, and thus a starting point to construct a theory, rather than a theory itself. Further research to fill in the framework's mathematical details would have to be supplied by mathematicians.

References

Sewell, Kip K. (2023). *Forever Finite: The Case Against Infinity*. Alexandria, VA: Rond Books. See the following:

[1] § 1.1: pp. 19–24.
[2] § 1.6: Figure 1.4, p. 46.
[3] § 1.3: p. 26.
[4] § 1.3.1: p. 26.
[5] § 1.3.2: pp. 27–28.
[6] § 1.3.1: p. 27.
[7] § 3.3.4, § 3.6.2, § 13.2.1: pp. 112-113, 122-123, 348-350.
[8] § 1.3.2: pp. 27–29; Chapter 8.
[9] § 3.2: pp. 103–108.
[10] §§ 3.2.7– 3.2.8: pp. 107–108.
[11] § 3.8: pp. 126–127.
[12] § 3.13.1: p. 135.
[13] § 5.4.8: pp. 188–191.
[14] § 5.4.8: pp. 190–191.
[15] § 5.3.1: pp. 171–173.
[16] § 5.4.6, § 5.4.8: pp. 187, 190.
[17] See Chapter 8, especially §§ 8.1.7– 8.1.10, including Table 2, p. 236.
[18] § 1.3.3: pp. 29–30.
[19] § 1.3.3: p. 30.
[20] § 1.3.1: p. 27.
[21] § 6.1: p. 195.
[22] § 6.1.2: p. 196.
[23] § 6.3: p. 199.
[24] § 6.3: p. 199.
[25] pp. 627–628.
[26] pp. 627–628.
[27] § 5.4.11: p. 192.
[28] § 25.1.3: p. 661.
[29] § 1.4: p. 36.
[30] § 1.4: pp. 33–37.
[31] § 1.4: pp. 37–38.
[32] § 1.4: p. 38.
[33] § 1.4.1: pp. 38–39.
[34] § 1.4: p. 38.
[35] § 1.4.2: p. 40.

[36] § 1.4.2: p. 39.
[37] § 1.5: pp. 43–44.
[38] § 1.5: p. 45.
[39] § 1.5: pp. 44–45.
[40] § 1.5: p. 45.
[41] § 1.6: p. 46.
[42] § 1.7, § 1.7.6, § 27.3: pp. 47, 50, 707.
[43] § 1.7.1, § 27.1: pp. 47, 701–703.
[44] § 1.7.1: p. 47.
[45] § 1.7.1, § 26.1.1: pp. 47, 685–686.
[46] § 26.1.1: pp. 685–686.
[47] § 1.7.1: p. 47.
[48] § 1.7.1: p. 47.
[49] § 5.3.2: pp. 173–174.
[50] § 1.1.2: pp. 21–23. The issue of infinite circles is addressed in § 18.4.2: pp. 500–501.
[51] § 27.2.6: p. 707.
[52] § 10.7: Figure 10.2, p. 304.
[53] § 11.7: Figure 11.6, p. 324.
[54] § 12.7: pp. 338–343.
[55] § 23.1.1, § 23.2.1, § 23.2.3, § 23.3.1, § 23.6: pp. 629, 634, 636–638, 643.
[56] § 25.2.9: pp. 670–673.
[57] Ch. 13 intro and § 13.1: pp. 345–347.
[58] § 13.1: pp. 345–347.
[59] Ch. 14.
[60] § 14.11.3: pp. 425–426.
[61] § 14.7.7: p. 415.
[62] Ch. 14.
[63] § 17.2.2: pp. 478–479.
[64] §§ 17.3–17.4: pp. 479–490.
[65] Chs. 18–21.
[66] Ch. 23 intro.
[67] § 8.1.8, § 23.5: pp. 233–234, 642.
[68] Ch. 23 intro.
[69] § 23.6: pp. 643–644.
[70] Ch. 24 intro.
[71] § 18.4, § 24.1.1: pp. 497, 645-646.
[72] § 24.1.3: p. 647.
[73] § 24.1.2: p. 646.
[74] § 24.1.4: pp. 647–648.
[75] § 24.2.2, § 24.4: pp 651–652.
[76] Ch. 26.
[77] Ch. 26 intro, p. 683.
[78] § 25.4: p. 681.
[79] § 26.1.1: pp. 685–686.
[80] § 25.2.12, § 26.1: pp. 676, 683–684.
[81] Ch. 26 intro: p. 683.

[82] § 25.4.2: p. 682.
[83] § 22.1.2, § 26.1.1: pp. 615, 685–686.
[84] § 26.
[85] § 26.1.2: p. 686.
[86] § 26.2: p. 687–688.
[87] § 26.6.2: pp. 696–697.
[88] § 27.3: p. 707.
[89] § 27.4: pp. 708–713.
[90] § 27.4.1: pp. 708–709.
[91] § 27.4.2: p. 709.
[92] § 27.4.3: p. 709.
[93] § 27.4.3: pp. 709–710.
[94] § 27.4.4: p. 711.
[95] § 26.6.2, § 27.4.5: pp. 696–697, 711–712.
[96] § 27.4.6: pp. 712–713.
[97] See Appendix (expanded edition only): pp. 717–757.